

New Q-ball Solutions in Gauge-Mediation, Affleck-Dine Baryogenesis and Gravitino Dark Matter

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Affleck-Dine (AD) baryogenesis along a $d = 6$ flat direction in gauge-mediated supersymmetry-breaking (GMSB) models can produce unstable Q-balls which naturally have field strength similar to the messenger scale. In this case a new kind of Q-ball is formed, intermediate between gravity-mediated and gauge-mediated type. We study in detail these new Q-ball solutions, showing how their properties interpolate between standard gravity-mediated and gauge-mediated Q-balls as the AD field becomes larger than the messenger scale. It is shown that E/Q for the Q-balls can be greater than the nucleon mass but less than the MSSM-LSP mass, leading to Q-ball decay directly to Standard Model fermions with no MSSM-LSP production. More significantly, if E/Q is greater than the MSSM-LSP mass, decaying Q-balls can provide a natural source of non-thermal MSSM-LSPs, which can subsequently decay to gravitino dark matter without violating nucleosynthesis constraints. The model therefore provides a minimal scenario for baryogenesis and gravitino dark matter in the gauge-mediated MSSM, requiring no new fields.

PACS numbers: 12.60.Jv, 98.80.Cq, 95.35.+d

I. INTRODUCTION

Affleck-Dine baryogenesis is a particularly simple way to generate the baryon asymmetry in the MSSM [1]. However, in GMSB models, AD baryogenesis is severely constrained by Q-ball formation. For field strengths greater than the messenger mass, the potential is approximately flat. Q-balls forming in a flat potential have energy-per-charge $E/Q \propto Q^{-1/4}$, therefore for Q large enough, E/Q is less than the nucleon mass m_n . In this case, as a result of baryon number conservation, Q-balls cannot decay and would exist in the Universe at present [2]. Q-balls absorbed by neutron stars would destabilize the stars and so are observationally ruled out [3, 4]. Thus for AD baryogenesis to succeed in GMSB models, the Q-balls must be unstable.

In the MSSM with R-parity conservation, the lowest dimension flat directions which can support AD baryogenesis or leptogenesis are the $d = 4$ $(H_u L)^2$ and $d = 6$ $(u^c d^c d^c)^2$ directions. $d = 4$ AD leptogenesis is a high reheating temperature (T_R) variant of AD baryogenesis, with $T_R \sim 10^8$ GeV. In contrast, $d = 6$ AD baryogenesis is a low reheating temperature variant, with $T_R \lesssim 100$ GeV, where the low reheating temperature is necessary to dilute the larger initial baryon density of the AD condensate.

$d = 4$ AD leptogenesis along the $(H_u L)^2$ direction is possible because the AD condensate decays and thermalizes at a temperature greater than that of the electroweak phase transition, T_{EW} . In this case the lepton asymmetry is transferred to a baryon asymmetry via $B + L$ -violating sphaleron fluctuations¹. $d = 6$ AD baryogenesis is dynamically more complex. The AD condensate is unstable with respect to perturbations and fragments into condensate lumps, which subsequently evolve into Q-balls [2, 5–11]. The large field and baryonic charge of the $d = 6$ Q-balls protects them from thermalization and leads to a low decay temperature $T_d \lesssim 1$ GeV [5, 6]. As a result, $d = 6$ Q-balls can be a natural source of non-thermal NLSPs in the MSSM [6, 12]. The formation of large charge Q-balls also protects $d = 6$ AD baryogenesis from the effect of additional lepton number violation in extensions of the MSSM, which could washout the baryon asymmetry from $d = 4$ AD leptogenesis. More importantly, late-decaying Q-balls could provide a minimal scenario for gravitino dark matter in the MSSM, which requires a natural source of non-thermal NLSPs.

In this paper we will focus on the case of $d = 6$ AD baryogenesis. In [13] we considered the case of AD baryogenesis along the $d = 6$ $(u^c d^c d^c)^2$ of the MSSM in GMSB. We showed that if the gravitino mass is large enough, not much smaller than 1 GeV, then it is possible for the AD field at the onset of baryogenesis to be similar to the messenger scale when the condensate fragments. In this case, since the Q-balls are not forming on the logarithmic plateau of the GMSB flat direction potential, they can have $E/Q > m_n$ and so can decay before the era of nucleosynthesis, avoiding the problem of stable Q-balls.

In addition, late-decaying Q-balls open up new possibilities for gravitino dark matter in the gauge-mediated MSSM. Decay of thermal relic MSSM-LSPs² as a source of gravitino dark matter appears to be generally ruled out by BBN constraints [14]³. (See

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¹ Even if Q-balls form along the $d = 4$ direction, they have a small enough charge to decay at $T > T_{EW}$.

² MSSM-LSP refers to the LSP of the MSSM sector i.e. excluding gravitinos and RH sneutrinos. The true LSP is assumed to be the gravitino. The MSSM-LSP is also the NLSP except in the case of a RH sneutrino NLSP.

³ Early studies of the effect of decaying particles on element abundances can be found in [15].

[16] for a discussion of possible exceptions.) Therefore a non-thermal source of MSSM-LSPs is required. The MSSM-LSPs should be produced below the freeze-out temperature of the MSSM-LSP. In this case, non-thermal stau or sneutrino MSSM-LSPs can produce gravitino dark matter and remain consistent with BBN if the MSSM-LSP mass is greater than around 300 GeV and $m_{3/2} \lesssim 1$ GeV (see Figs.14 and 16 of [14]), while a bino MSSM-LSP of mass greater than about 300 GeV is consistent with BBN if $m_{3/2} \lesssim 2 \times 10^{-2}$ GeV, with the upper bound on $m_{3/2}$ increasing with increasing bino mass (Figs. 9 and 10 of [14]). Since $d = 6$ Q-balls typically decay below the MSSM-LSP freeze-out temperature, they can provide a natural source of non-thermal MSSM-LSPs in the gauge-mediated MSSM.

Alternatively, if RH sneutrino NLSPs have the right properties, Q-ball decay to RH sneutrinos might produce gravitino dark matter while remaining consistent with BBN even if $m_{3/2} > 1$ GeV. In the case where only positive charged Q-balls result from condensate fragmentation, B conservation combined with R-parity conservation implies that $m_{3/2} \approx 2 \text{ GeV}^4$. This is self-consistent with the large gravitino mass required to have unstable Q-balls. A successful RH sneutrino NLSP scenario with $m_{3/2} \approx 2$ GeV requires that the MSSM-LSPs from Q-ball decay can decay to RH sneutrinos before nucleosynthesis and that the RH sneutrinos can decay to gravitinos without violating nucleosynthesis and free-streaming constraints [18], which might be achieved if the RH neutrinos have enhanced Yukawa couplings via the see-saw mechanism [19].

The analysis of Q-ball decay in [13] was based on the assumption that the GMSB Q-balls with $\phi(0) \sim M_m$, where $\phi(0)$ is the field strength at the centre of the Q-ball, could be approximated by gravity-mediated-type Q-balls. However, Q-balls in the transition region between the $|\Phi|^2$ potential at $|\Phi|/M_m \ll 1$ and the approximately constant (logarithmic) potential at $|\Phi|/M_m \gg 1$ will be of a new type⁵, interpolating between gravity-mediated type with E/Q approximately constant and gauge-mediated type with $E/Q \propto Q^{-1/4}$. Since the value of E/Q determines the decay properties of the Q-balls, in particular their decay temperature and whether MSSM-LSPs are produced in Q-ball decay, it is important to understand these Q-balls in detail. In this paper we will study the Q-ball solutions in the transition region as a function of $\phi(0)/M_m$.

The paper is organized as follows. In Section 2 we discuss the flat-direction potential which models a generic GMSB flat-direction. In Section 3 we derive the equation for the Q-ball solutions. In Section 4 we show that the Q-ball solutions have a useful scaling property, broken only by small gravity-mediated contributions. In Section 5 we present our results for the properties of the Q-balls as a function of $\phi(0)/M_m$. In Section 6 we discuss the implications of our results for AD baryogenesis and gravitino dark matter. In Section 7 we present our conclusions.

II. FLAT-DIRECTION POTENTIAL IN GMSB

GMSB models are based on SUSY breaking in a hidden sector which is transmitted to the MSSM via vector pairs of messenger fields carrying SM gauge charges. The messenger superfield scalar components acquire SUSY breaking mass splittings from their interaction with the hidden sector. The messengers then induce masses for MSSM gauginos at 1-loop and soft SUSY breaking scalar mass squared terms at 2-loops. A key relation is that between the gravitino mass and the messenger mass, given by [13]

$$M_m \approx \frac{g^2}{16\pi^2} \frac{\sqrt{3}\kappa m_{3/2} M_p}{m_s}, \quad (1)$$

where g is the gauge coupling of the messengers, κ is the superpotential coupling of the messengers to the SUSY breaking field and m_s is the soft SUSY breaking scalar mass. Therefore, assuming minimal SUGRA,

$$M_m \approx 5 \times 10^{13} g^2 \kappa \left(\frac{m_{3/2}}{2 \text{ GeV}} \right) \left(\frac{100 \text{ GeV}}{m_s} \right) \text{ GeV}. \quad (2)$$

Therefore large gravitino masses are required for large M_m .

Q-balls form in a scalar field theory with a global $U(1)$ symmetry, when the potential $V(|\Phi|)$ is flatter than $|\Phi|^2$, corresponding to an attractive interaction between the scalar particles. In the case of GMSB models, the form of the flat direction potential in the region of $|\Phi|$ of interest is [13, 21]

$$V(\Phi) = m_s^2 M_m^2 \ln^2 \left(1 + \frac{|\Phi|}{M_m} \right) \left(1 + K \ln \left(\frac{|\Phi|^2}{M_m^2} \right) \right) + m_{3/2}^2 \left(1 + \hat{K} \ln \left(\frac{|\Phi|^2}{M_m^2} \right) \right) |\Phi|^2. \quad (3)$$

⁴ In the context of gravity-mediated SUSY breaking, Q-ball decay to 2 GeV axino LSPs was proposed in [17] in order to satisfy the required LSP mass.

⁵ We should distinguish these Q-balls from another Q-ball solution which interpolates between gauge and gravity-mediated Q-balls [20]. These were derived for the case of large AD field, where the gravity-mediated contribution to the potential comes to dominate the gauge-mediated contribution.

In this we assume $|\Phi|$ is small enough that the $U(1)$ -violating A-terms and the non-renormalizable potential terms of the full GMSB potential can be neglected. The former condition is essential for the existence of Q-balls. The first term in Eq. (3) is due to GMSB with messenger mass M_m . The factor multiplying this takes into account 1-loop radiative corrections due to gaugino loops once $|\Phi| \lesssim M_m$, with $K \approx -(0.01 - 0.1)$ [5–7]. The second term is due to gravity-mediated SUSY breaking including the 1-loop correction term \hat{K} . (For simplicity we set $\hat{K} = K$.) In the limit $|\Phi|/M_m \ll 1$, the potential Eq. (3) tends towards a gravity-mediated-type potential of the form

$$V(|\Phi|) \approx m_s^2 \left(1 + K \ln \left(\frac{|\Phi|^2}{M_m^2} \right) \right) |\Phi|^2 + m_{3/2}^2 \left(1 + \hat{K} \ln \left(\frac{|\Phi|^2}{M_m^2} \right) \right) |\Phi|^2, \quad (4)$$

while in the limit $|\Phi|/M_m \gg 1$ the potential has a slow logarithmic growth with $|\Phi|$ (the GMSB ‘plateau’) plus a small contribution from gravity-mediated SUSY breaking,

$$V(|\Phi|) \approx m_s^2 M_m^2 \ln^2 \left(\frac{|\Phi|}{M_m} \right) + m_{3/2}^2 \left(1 + \hat{K} \ln \left(\frac{|\Phi|^2}{M_m^2} \right) \right) |\Phi|^2. \quad (5)$$

(The term proportional to K is a small correction to the potential in this case and may be neglected.) Q-ball solutions at $\phi(0)/M_m \ll 1$ will have the form of gravity-mediated Q-balls, with a Gaussian profile $\phi(r)$ and constant $E/Q \approx m_s$. At $|\Phi|/M_m \gg 1$, the potential is not exactly constant but the Q-balls are expected to be similar to those for a perfectly constant flat direction potential, which have $E/Q \propto Q^{-1/4}$ and therefore have a suppressed E/Q value at large enough Q .

In the case of $d = 6$ flat directions in GMSB, when $m_{3/2} = 2$ GeV, which is consistent with gravitino dark matter from Q-ball decay in the case where only positively charged Q-balls form, the onset of oscillations of the Φ field occurs when $|\Phi|$ is close to or somewhat larger than the messenger mass. The precise value at which oscillations begin is sensitive to the non-renormalizable B-violating superpotential operator which lifts the flat direction. This is assumed to be of the form $W = \Phi^6/6!\tilde{M}^3$, where \tilde{M} is assumed close to the Planck scale, $\tilde{M} = (0.1 - 1)M_{Pl}$. There is then a period of time during which quantum fluctuations of the AD field grow and $|\Phi|$ decreases due to the expansion of the Universe, eventually breaking up the condensate into fragments carrying large baryon number. As a result, for typical parameters in Eq. (3), the value of $|\Phi|/M_m$ in the fragments is typically in the transition region between $|\Phi|/M_m \ll 1$ and $|\Phi|/M_m \gg 1$. These fragments subsequently evolve into Q-balls. The evolution of the fragments into Q-balls is determined by the non-linear dynamics of Φ , and may result in the production of purely positive or both positive and negative charged Q-balls [22].

The value of E/Q is crucial for determining the decay properties of the Q-balls. Each unit of Q corresponds to an R-parity odd squark absorbed by the Q-ball. Therefore $|\Delta Q| = 1$ corresponds to a change of sign of the R-parity of the Q-ball, and so a MSSM-LSP must be emitted by the Q-ball. Thus B conservation combined with R-parity conservation requires that $n_B = 3n_\chi$ from Q-ball decay, where χ denotes the MSSM-LSP. (In the following the global charge Q of Φ is normalized to 1, so that $B = Q/3$.) To have Q-ball decay to MSSM-LSPs which is not kinematically suppressed, we must therefore have $E/Q > m_\chi$, otherwise the decay would have to occur through $|\Delta Q| = 2$ processes to R-parity even final states of SM quarks and leptons. This process may be thought of as squarks in the Q-ball annihilating with each other to quark pairs, rather than individually decaying to a quark plus MSSM-LSP. This is a new mode of Q-ball decay which allows a new scenario for AD baryogenesis in GMSB models, with Q-balls decaying at low temperature to baryon number without accompanying MSSM-LSPs. This possibility has also been noted in [23], where it plays a central role in an alternative model for gravitino dark matter from Q-ball decay.

III. Q-BALL SOLUTIONS FOR GMSB FLAT DIRECTIONS

Q-balls are minimum energy configurations for a fixed global charge Q . The solutions are obtained by introducing a Lagrange multiplier ω for the conserved charge and minimizing the functional

$$E_\omega(\Phi, \dot{\Phi}, \omega) = E + \omega \left(Q - \int d^3x \rho_Q \right), \quad (6)$$

where

$$E = \int d^3x |\dot{\Phi}|^2 + |\nabla \Phi|^2 + V(|\Phi|) \quad (7)$$

and

$$\rho_Q = i \left(\dot{\Phi}^\dagger \Phi - \Phi^\dagger \dot{\Phi} \right) \quad (8)$$

The minimum energy solutions have the form

$$\Phi = \frac{\varphi(r)}{\sqrt{2}} e^{i\omega t}, \quad (9)$$

where the Q-ball profile $\varphi(r)$ is given by the solution of

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{2}{r} \frac{\partial \varphi}{\partial r} = \frac{\partial V}{\partial \varphi} - \omega^2 \varphi, \quad (10)$$

with boundary conditions $\varphi'(r) = 0$ as $r \rightarrow 0$ and $\varphi(r) \rightarrow 0$ as $r \rightarrow \infty$.

For the GMSB potential Eq. (3), the equation for the Q-ball profile is

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{2}{r} \frac{\partial \varphi}{\partial r} = m_s^2 M_m^2 \frac{\sqrt{2} \ln \left(1 + \frac{\varphi}{\sqrt{2} M_m} \right)}{\left(1 + \frac{\varphi}{\sqrt{2} M_m} \right)} \left(1 + K \ln \left(\frac{\varphi}{2 M_m} \right) \right) + \frac{2K}{\varphi} \ln^2 \left(1 + \frac{\varphi}{\sqrt{2} M_m} \right) + m_{3/2}^2 \varphi \left(1 + K \ln \left(\frac{\varphi}{2 M_m} \right) \right) - \omega^2 \varphi. \quad (11)$$

Our method to solve Eq. (11) is to fix the value of $\varphi(0)/M_m$ and integrate Eq. (11) with the boundary condition $\varphi'(0) = 0$. We then vary ω until $\varphi(r) \rightarrow 0$ as $r \rightarrow \infty$. This determines the Q-ball profile $\varphi(r)$ and the value of ω for the given value of $\varphi(0)/M_m$.

The total charge of the Q-ball is then

$$Q = \int_0^\infty 4\pi r^2 \omega \varphi(r)^2 dr, \quad (12)$$

while the total energy is

$$E = \int_0^\infty 4\pi r^2 \left[\frac{1}{2} \left(\frac{\partial \varphi}{\partial r} \right)^2 + V(\varphi) + \frac{\omega^2 \varphi^2}{2} \right] dr. \quad (13)$$

Using these can compute E/Q for the Q-ball. The radius of the Q-ball presented in our tables is defined to be the radius within which 90% of the total energy is found.

In the limit $\varphi(0)/M_m \ll 1$, the Q-balls are of gravity-mediated type, in which case an exact solution exists. For the potential

$$V(\Phi) = m_s^2 \left(1 + K \ln \left(\frac{|\Phi|^2}{M_m^2} \right) \right) |\Phi|^2, \quad (14)$$

the solution for $\varphi(r)$ is

$$\varphi(r) = \varphi(0) e^{-r^2/R^2} \quad (15)$$

where

$$R^2 = \frac{2}{|K|m_s^2}; \quad \omega^2 = \omega_o^2 + m_s^2(1+K); \quad \omega_o^2 = 3|K|m_s^2 + K \ln \left(\frac{\varphi(0)^2}{M_m^2} \right).$$

Then $E/Q \approx \omega \approx m_s$ when $|K| \ll 1$, independent of $\varphi(0)/M_m$. In the opposite limit, $\varphi(0)/M_m \gg 1$, if the potential is treated as constant then $E/Q \propto Q^{-1/4}$.

A key quantity for AD baryogenesis is the Q-ball decay temperature. Q-ball decay to fermions has an upper bound from Pauli blocking. On cosmological time scales this will generally be saturated, therefore [24]

$$T_d = \left(\frac{\omega^3 R^2 M_{Pl}}{48\pi k_T Q} \right)^{1/2}, \quad (16)$$

where $k_T = (g(T)\pi^2/90)^{1/2}$ and $g(T)$ is the effective number of relativistic degrees of freedom, with the expansion rate during radiation domination given by $H = k_T T^2/M_{Pl}$.

IV. SCALING PROPERTIES OF THE Q-BALL SOLUTIONS

The solutions for any m_s and M_m can be obtained by rescaling the solution of Eq. (11), up to small corrections from the gravitino mass, which breaks scale-invariance. This can be seen by expressing the Q-ball equation in terms of $\hat{\phi} = \phi/M_m$, $\hat{r} = rm_s$ and $\hat{\omega} = \omega/m_s$. Eq. (11) then becomes

$$\frac{\partial^2 \hat{\phi}}{\partial \hat{r}^2} + \frac{2}{\hat{r}} \frac{\partial \hat{\phi}}{\partial \hat{r}} = \frac{\sqrt{2} \ln \left(1 + \frac{\hat{\phi}}{\sqrt{2}} \right)}{\left(1 + \frac{\hat{\phi}}{\sqrt{2}} \right)} \left(1 + K \ln \left(\frac{\hat{\phi}}{2} \right) \right) + \frac{2K}{\hat{\phi}} \ln^2 \left(1 + \frac{\hat{\phi}}{\sqrt{2}} \right) + \left(\frac{m_{3/2}}{m_s} \right)^2 \hat{\phi} \left(1 + K \ln \left(\frac{\hat{\phi}}{2} \right) \right) - \hat{\omega}^2 \hat{\phi}. \quad (17)$$

In the limit $m_{3/2} \rightarrow 0$ this has a unique solution for a given K and $\phi(0)/M_m$. For a given m_s and M_m this can then be transformed to the physical solution using $\phi(r) = M_m \hat{\phi}(m_s r)$. The energy and charge of the Q-ball scale as $E \propto M_m^2/m_s$ and $Q \propto M_m^2/m_s^2$, as can be seen by expressing Eq. (12) and Eq. (13) in terms of $\hat{\phi}$, \hat{r} and $\hat{\omega}$. Therefore $E/Q \propto m_s$ and is independent of M_m . These properties are confirmed by the results in Tables 1-3. Using these scaling properties and the results in the Tables, we can obtain accurate estimates of the Q-ball properties for any m_s and M_m , up to small corrections of order $(m_{3/2}/m_s)^2$. In particular, applying this scaling to the Q-ball decay temperature gives $T_d \propto m_s^{3/2}/M_m$.

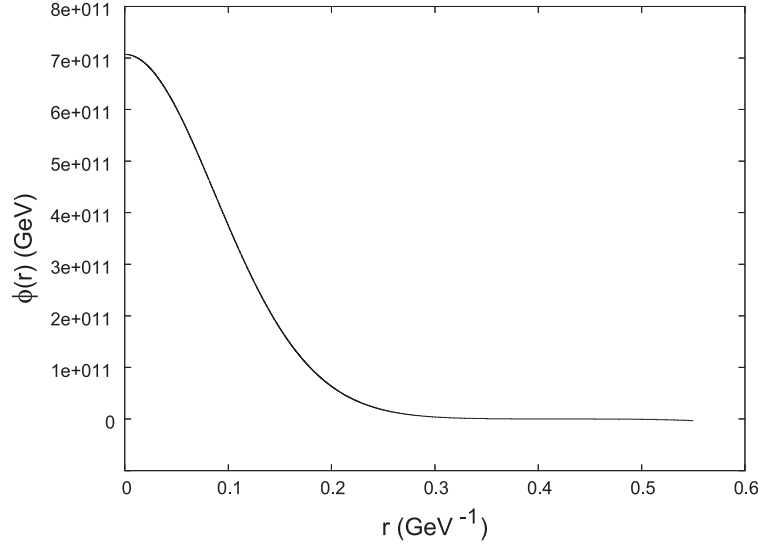


FIG. 1: Q-Ball Solution for $\phi(0)/M_m = 0.01$, $K = -0.01$, $m_{3/2} = 2$ GeV, $M_m = 10^{14}$ GeV and $m_s = 100$ GeV.

V. RESULTS

In Figures 1-3 we show the Q-ball profile $\phi(r)/M_m$ for the cases $\phi(0)/M_m = 0.1, 1$ and 100 , when $K = -0.01$ and $m_s = 100$ GeV. (We fix $M_m = 10^{14}$ GeV and $m_{3/2} = 2$ GeV throughout.) For $\phi(0)/M_m = 0.1$ the Q-ball closely matches the Gaussian profile expected for a gravity-mediated-type Q-ball. For $\phi(0)/M_m \gg 1$ the profile deviates strongly from Gaussian, with a much wider profile for a given $\phi(0)/M_m$.

In Figures 4 and 5 we show the Q-ball radius as a function of $\phi(0)/M_m$ for $K = -0.01$ and $K = -0.1$. This shows the Q-ball radius decreasing slightly as $\phi(0)/M_m$ approaches 1, then increasing almost linearly with $\phi(0)/M_m$ as it becomes larger than 1.

In Figure 6 we show E/Q as a function of $\phi(0)/M_m$ for $K = -0.01$ and $m_s = 100$ GeV. (The corresponding value of ω for the Q-ball solutions is shown in Figure 7.) A close-up of E/Q as a function of $\phi(0)/M_m$ around $\phi(0)/M_m \sim 1$ is shown in Figure 8. E/Q decreases from $E/Q \approx m_s$ to $E/Q \approx 0.15m_s$ as $\phi(0)/M_m$ increases from 0.01 to 100. Therefore, even for large $\phi(0)/M_m$, E/Q can be larger than the nucleon mass and Q-balls can be unstable. Even larger values of $\phi(0)/M_m$ would be allowed with larger m_s . Thus $d = 6$ AD baryogenesis can easily produce unstable Q-balls, even though E/Q can be significantly suppressed relative to m_s .

In Figure 9 we show E/Q as a function of Q . For $\ln(Q) \lesssim 60$, corresponding to $\phi(0)/M_m \lesssim 1$, E/Q becomes independent of Q , as expected for a gravity-mediated-type Q-ball. At larger Q we find an almost constant negative slope, corresponding to

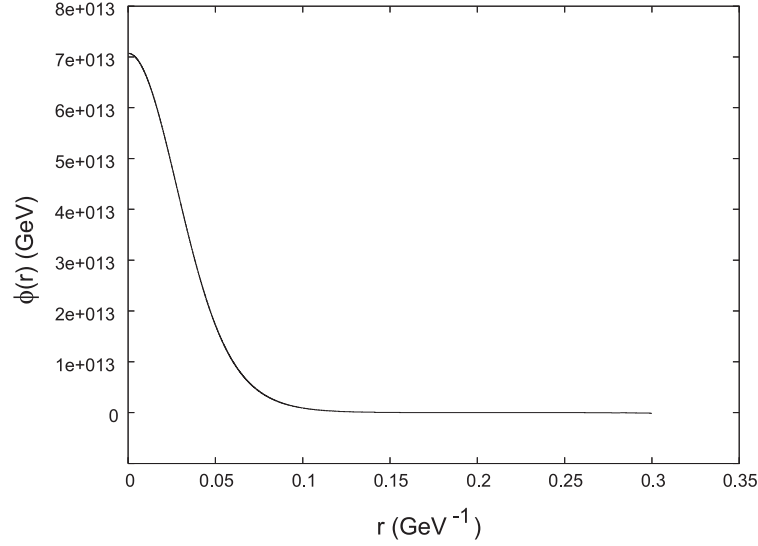


FIG. 2: Q-Ball Solution for $\phi(0)/M_m = 1$, $K = -0.01$, $m_{3/2} = 2$ GeV, $M_m = 10^{14}$ GeV and $m_s = 100$ GeV.

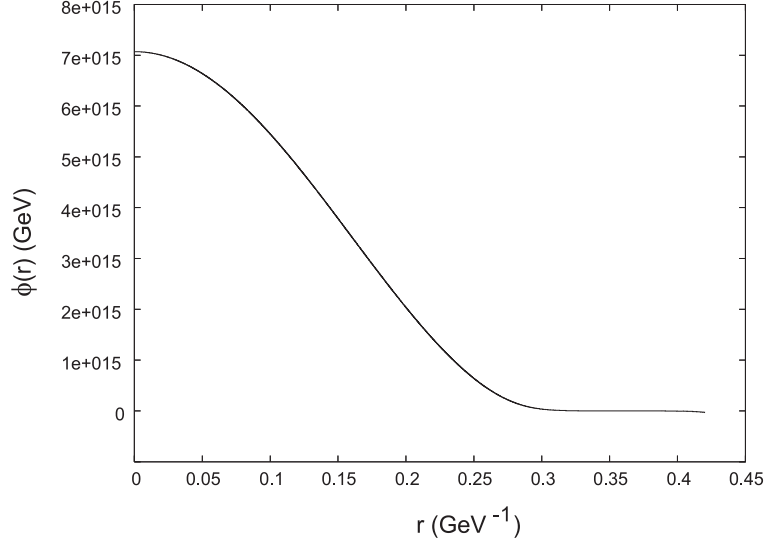


FIG. 3: Q-Ball Solution for $\phi(0)/M_m = 100$, $K = -0.01$, $m_{3/2} = 2$ GeV, $M_m = 10^{14}$ GeV and $m_s = 100$ GeV.

$E/Q \propto Q^{-n}$ with $n = 0.22$. This differs slightly from the conventional value for gauge-mediated-type Q-balls, $n = 0.25$. However, $n = 0.25$ assumes a completely flat potential, whereas in our case there is a squared logarithmic potential.

In Tables 1-3 we give the numerical properties of the Q-ball solutions as a function of $\phi(0)/M_m$ for the cases $K = -0.01$, $m_s = 100$ GeV (Table 1), $K = -0.01$, $m_s = 1$ TeV (Table 2) and $K = -0.1$, $m_s = 100$ GeV (Table 3). In particular, we show Q and T_d as a function of $\phi(0)/M_m$. The tables assume $M_m = 10^{14}$ GeV; the results can be scaled to other values of m_s and M_m by using the scaling properties discussed in the previous section. We see that a large baryonic charge is typical for the Q-balls, with $Q \gtrsim 10^{21}$ for the examples given in the tables. As $\phi(0)/M_m$ increases, the charge rapidly increases. As a result $T_d \propto Q^{-1/2}$ rapidly decreases. For the case $M_m = 10^{14}$ GeV and $m_s = 100$ GeV in Table 1, T_d becomes less than the nucleosynthesis bound ~ 1 MeV once $\phi(0)/M_m \gtrsim 1$. Smaller values of M_m and larger values of m_s allow larger $\phi(0)/M_m$ to be compatible with $T_d \gtrsim 1$ MeV, since T_d scales as $T_d \propto m_s^{3/2}/M_m$. So $M_m = 10^{13}$ GeV and $m_s = 500$ GeV enhances T_d by a factor 110, in which case $\phi(0)/M_m$ can be as large as 20. However, in general there is an upper limit on $\phi(0)/M_m$ from the Q-ball decay temperature. This is an important constraint on the model. Comparing Tables 1 and 3, we see that increasing $|K|$ from 0.01 to 0.1 also has a small effect on T_d , increasing T_d by a factor of 2 at $\phi(0)/M_m \lesssim 1$, less so at larger $\phi(0)/M_m$.

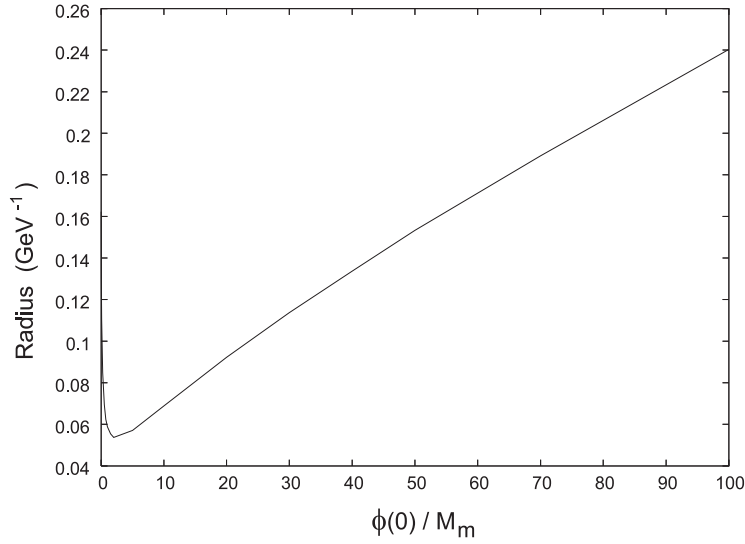


FIG. 4: The variation of the radius of the Q-ball for $K = -0.01$, $m_{3/2} = 2$ GeV, $M_m = 10^{14}$ GeV and $m_s = 100$ GeV.

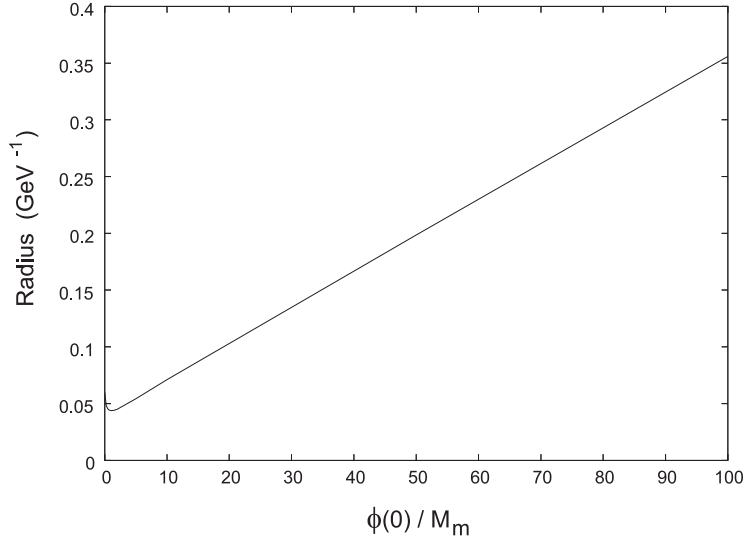


FIG. 5: The variation of the radius of the Q-ball for $K = -0.1$, $m_{3/2} = 2$ GeV, $M_m = 10^{14}$ GeV and $m_s = 100$ GeV.

VI. CONSEQUENCES FOR AD BARYOGENESIS AND GRAVITINO DARK MATTER IN GMSB

There are two issues facing AD baryogenesis in GMSB models: (i) are the Q-balls sufficiently unstable to decay before BBN and (ii) if NLSPs are produced in Q-ball decay, can they decay to gravitinos (and, in particular, gravitino dark matter) without causing problems for BBN.

A new possibility in GMSB is that E/Q can be less than the MSSM-LSP mass but greater than the nucleon mass. This is in contrast to the case of gravity-mediated Q-balls, where E/Q is approximately equal to the mass of the squarks forming the Q-ball. Since squarks generally cannot be the MSSM-LSP (which we denote by χ), in this case $E/Q > m_\chi$ and so the Q-balls can always decay to MSSM-LSPs. In GMSB models, on the other hand, if $E/Q < m_\chi$ then decay to MSSM-LSPs is kinematically excluded. Q-balls can then only decay by a process which is related to annihilation of the squarks making up the Q-ball, producing only SM fermions. As a result, Q-ball decay will generally occur without problems for BBN due to subsequent MSSM-LSP decay. Therefore $d = 6$ AD baryogenesis can be generally consistent with BBN even in the case of large gravitino mass $\gtrsim 1$ GeV, which is severely constrained by BBN [14]. Thermal relic MSSM-LSPs could still be a problem for BBN in this case, for example thermal relic bino dark matter is essentially ruled out for $m_{3/2} > 1$ GeV (Fig.7 of [14]). However, their density

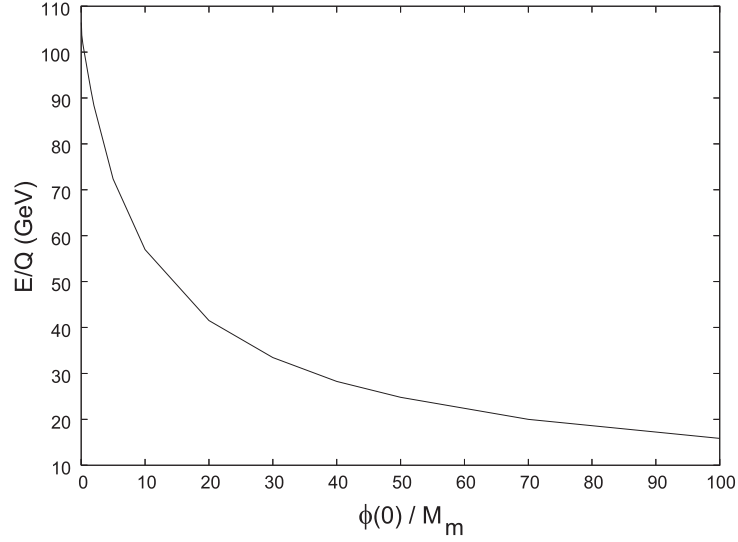


FIG. 6: The variation of E/Q as $\phi(0)/M_m$ grows, with $K = -0.01$, $m_{3/2} = 2$ GeV, $M_m = 10^{14}$ GeV and $m_s = 100$ GeV.

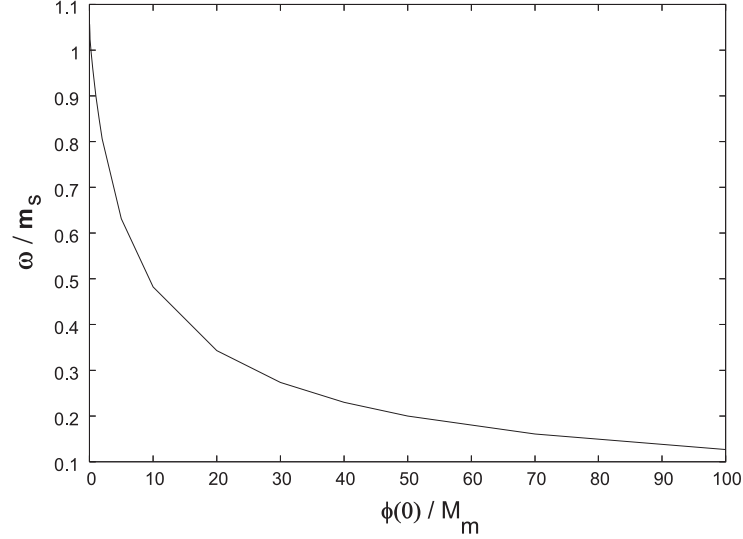


FIG. 7: ω/m_s vs. $\phi(0)/M_m$ for $K = -0.01$, $M_m = 10^{14}$ GeV, $m_{3/2} = 2$ GeV and $m_s = 100$ GeV.

would be diluted by a factor $(T_R/T_\chi)^5$ if the reheating temperature is less than the MSSM-LSP freeze-out temperature.

In addition, $|\Phi|/M_m$ could be significantly larger than 1 when the condensate fragments. This would allow smaller messenger masses to be consistent with unstable Q-balls. However, as we have shown, there will be an upper limit on $|\Phi|/M_m$ from the decay temperature of the Q-balls, which decreases with $\phi(0)/M_m$ and eventually will drop the nucleosynthesis bound.

In general, decay of thermal relic MSSM-LSPs cannot produce gravitino dark matter, since their decay would violate BBN constraints [14]. In addition, since the reheating temperature of the $d = 6$ AD baryogenesis model is low, $T_R \lesssim 10$ GeV [13], gravitino dark matter cannot be generated by thermal scattering. Therefore an alternative dark matter candidate to the gravitino LSP is necessary when $E/Q < m_\chi$. The density of this candidate should not be diluted by the low reheating temperature. A natural possibility is the axion, whose density is effectively created at the QCD phase transition at low temperature.

In the case where $E/Q > m_\chi$, GMSB Q-balls can decay to MSSM-LSPs. In this case, Q-ball decay to non-thermal MSSM-LSPs may be compatible with BBN. This requires that $m_{3/2} \lesssim 1$ GeV [14]. In the case where the AD condensate evolves to purely positive charged Q-balls, the gravitino LSP mass must be $m_{3/2} \approx 2$ GeV in order to account for dark matter. This is slightly larger than the upper bound from BBN. If the MSSM-LSP density from Q-ball decay can be slightly enhanced relative to the baryon density, decreasing the gravitino mass to less than 1 GeV, Q-ball decay could provide a mechanism for gravitino

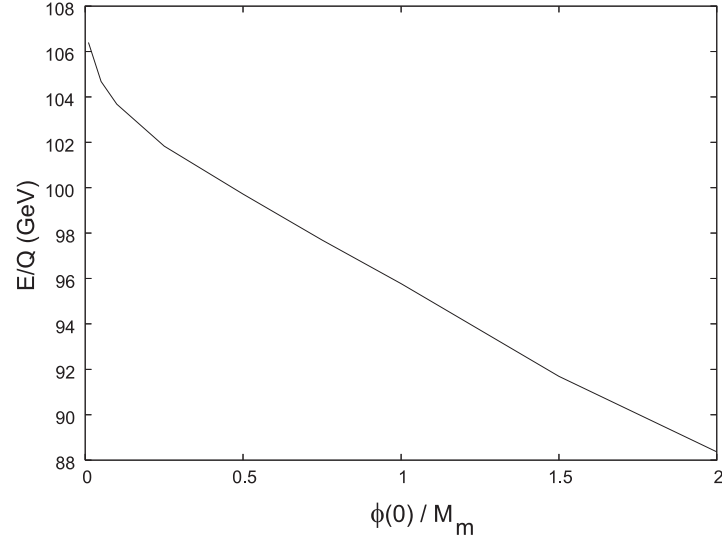


FIG. 8: The variation of E/Q as $\phi(0)/M_m$ grows, close-up on the transition region, $\phi(0)/M_m = 0.01$ to $\phi/M_m = 2$.

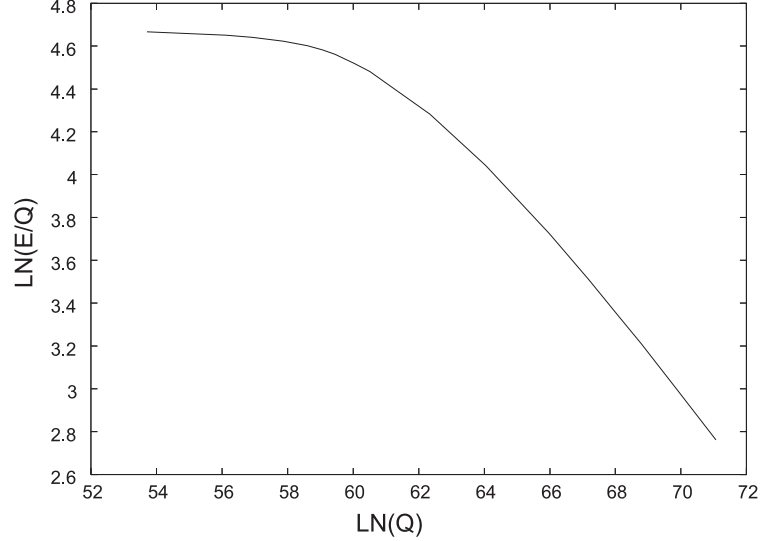


FIG. 9: $\text{LN}(E/Q)$ vs. $\text{LN}(Q)$ for $K = -0.01$, $M_m = 10^{14}$ GeV, $m_{3/2} = 2$ GeV and $m_s = 100$ GeV.

dark matter. This can be achieved if the AD condensate fragments to both positive and negative charged Q-balls. This has been observed in numerical simulations in the case where the original condensate fragments have more energy than Q-balls of the same charge [22], in which case the excess energy is converted into \pm Q-ball pairs. Such large energy fragments are natural in GMSB, since the AD condensate is strongly ellipitcal. In this case the decay of the positive and negative charged Q-balls will increase the number of MSSM-LSPs produced per baryon number from Q-ball decay, since the MSSM-LSPs from \pm Q-ball decay do not cancel. The necessary enhancement of the MSSM-LSP density is very modest, a factor of 2 being sufficient to have $m_{3/2} \lesssim 1$ GeV. Such an enhancement is very likely to occur, based on the numerical results of [22]. A non-thermal MSSM-LSP density consistent with BBN could then be produced if $1 \text{ MeV} < T_d < T_\chi$.

Therefore $d = 6$ AD baryogenesis could explain the baryon asymmetry and provide a natural source of non-thermal MSSM-LSPs and gravitino dark matter in the gauge-mediated MSSM. This can be achieved with just the MSSM, requiring no additional particles or interactions.

To see if this Q-ball decay scenario for gravitino dark matter can be realized, we need to consider the full process of AD baryogenesis for a given set of potential parameters. We first need to compute the reheating temperature and the energy and charge of the condensate fragments. From the charge of the fragments we can estimate the charge of the resulting Q-balls and

$\phi(0)/M_m$	ω/m_s	E (GeV)	Q	E/Q (GeV)	$\ln(E/Q)$	$\ln(Q)$	$Radius$ (GeV $^{-1}$)	T_d (GeV)
0.01	1.056834897	2.2440E+25	2.109E+23	1.0640E+02	4.667216272	53.70567104	0.1622	0.047177955
0.05	1.036387958	2.4590E+26	2.349E+24	1.0468E+02	4.650935244	56.11603194	0.1303	0.01102814
0.1	1.023327904	5.4890E+26	5.294E+24	1.0368E+02	4.641342172	56.92861634	0.1107	0.006123415
0.25	0.99574093	1.3880E+27	1.363E+25	1.0183E+02	4.623345895	57.87431548	0.0849	0.002809281
0.5	0.959374796	2.8080E+27	2.816E+25	9.9716E+01	4.602325234	58.59994476	0.06937	0.001510267
0.75	0.92784697	4.4120E+27	4.516E+25	9.7697E+01	4.581871641	59.07225397	0.06239	0.00102016
1.0	0.899333086	6.2800E+27	6.557E+25	9.5776E+01	4.562006985	59.44516051	0.05874	0.000760638
1.5	0.849117189	1.0920E+28	1.191E+26	9.1688E+01	4.518387773	60.04200571	0.05542	0.000488515
2.0	0.805890811	1.6780E+28	1.899E+26	8.8362E+01	4.481445362	60.50853985	0.05369	0.000346546
5.0	0.631466547	8.5910E+28	1.187E+27	7.2376E+01	4.281871121	62.34122663	0.05704	0.00010214
10.0	0.481933605	3.7250E+29	6.540E+27	5.6961E+01	4.042361036	64.04767351	0.06900	3.50461E-05
20.0	0.342870238	1.8708E+30	4.509E+28	4.1488E+01	3.725415157	65.9784824	0.09220	1.07177E-05
30.0	0.273087898	5.0736E+30	1.519E+29	3.3401E+01	3.508594019	67.19301334	0.113720	5.1195E-06
40.0	0.2298456	1.0484E+31	3.711E+29	2.8251E+01	3.341136905	68.08628525	0.133740	2.97431E-06
50.0	0.199975	1.8642E+31	7.526E+29	2.4771E+01	3.209687665	68.79328222	0.153270	1.94239E-06
70.0	0.160802674	4.4862E+31	2.244E+30	1.9994E+01	2.995444763	69.88568799	0.189200	1.0013E-06
100.0	0.126520749	1.1600E+32	7.336E+30	1.5813E+01	2.760814215	71.0703029	0.240450	4.91113E-07

TABLE I: Q-ball properties for $m_s = 100$ GeV, $K = -0.01$ and $M_m = 10^{14}$ GeV.

$\phi(0)/M_m$	ω/m_s	E (GeV)	Q	E/Q (GeV)	$\ln(E/Q)$	$\ln(Q)$	$Radius$ (GeV $^{-1}$)	T_d (GeV)
0.01	1.056576074	2.25989E+24	2.12484E+21	1.06356E+03	6.969374913	49.10798346	0.01629	1.492192879
0.05	1.03614719	2.46530E+25	2.35726E+22	1.04583E+03	6.952568855	51.51437197	0.01305	0.348541733
0.10	1.023094326	5.50176E+25	5.30691E+22	1.03672E+03	6.943813575	52.32588179	0.0111	0.193861456
1.0	0.898900996	6.28241E+26	6.56997E+23	9.56231E+02	6.862999678	54.84196641	0.00586	0.023955186
10.0	0.481316943	3.11960E+28	6.53960E+25	4.77032E+02	6.167584066	59.44250333	0.00582	0.000934351
100.0	0.125039994	1.13925E+31	7.24925E+28	1.57154E+02	5.057227415	66.45328062	0.02408	1.53744E-05

TABLE II: Q-ball properties for $m_s = 1$ TeV, $K = -0.01$ and $M_m = 10^{14}$ GeV.

so determine the value of E/Q relative to the MSSM-LSP mass and whether the Q-balls decay before nucleosynthesis but after MSSM-LSP freeze-out. We also need to check that the messenger scale is plausibly compatible with the required gravitino mass from Eq. (1). A semi-analytical method to study AD baryogenesis and condensate fragmentation was given in [13]. It is beyond the scope of the present analysis to combine this method with the Q-ball solutions discussed here, but our initial estimates indicate the above scenario for $d = 6$ AD baryogenesis can be realized with reasonable assumptions for the parameters of the potential. We will present a complete analysis of the Q-ball decay model for baryogenesis and gravitino dark matter in a future study.

VII. CONCLUSIONS

We have presented a new type of Q-ball solution based on the form of flat-direction potential expected in $d = 6$ AD baryogenesis in the gauge-mediated MSSM. The solution interpolates between gravity-mediated-type Q-balls with constant E/Q at $\phi(0)/M_m \ll 1$ and gauge-mediated-type Q-balls with $E/Q \propto Q^{-1/4}$ at $\phi(0)/M_m \gg 1$. In general the gravitino mass should not be much smaller than 1 GeV in order to maintain the large messenger mass necessary for unstable Q-balls in $d = 6$ AD baryogenesis. The new Q-ball solutions can be unstable for $\phi(0)/M_m$ significantly larger than 1, with values $\mathcal{O}(100)$ only suppressing E/Q to about 15% of the AD scalar mass, which is generally much larger than the nucleon mass. Therefore taking into account the time delay for the AD condensate to fragment, the AD field can be significantly larger than the messenger scale at the onset of AD baryogenesis and still produce unstable Q-balls. This will allow smaller messenger masses and so smaller gravitino masses to be compatible with unstable Q-balls. However, there will be an upper limit on $\phi(0)/M_m$ from the Q-ball decay temperature and the nucleosynthesis bound.

$\phi(0)/M_m$	ω/m_s	E (GeV)	Q	E/Q (GeV)	$\ln(E/Q)$	$\ln(Q)$	Radius (GeV ⁻¹)	T_d (GeV)
0.01	1.478478948	1.8930E + 24	1.242E + 22	1.5242E + 02	5.026610075	50.87359503	0.05846	0.115940874
0.05	1.35882302	3.4749E + 25	2.477E + 23	1.4029E + 02	4.943687737	53.86650529	0.05881	0.023011682
0.1	1.2996538	1.0967E + 26	8.141E + 23	1.3471E + 02	4.903147927	55.05637016	0.05639	0.011384673
0.25	1.20573629	4.3940E + 26	3.466E + 24	1.2677E + 02	4.842408969	56.50504342	0.04830	0.004223059
0.5	1.115526781	1.2389E + 27	1.038E + 25	1.1935E + 02	4.78209829	57.60192311	0.04530	0.002036739
0.75	1.051094667	2.2920E + 27	2.018E + 25	1.1358E + 02	4.732488063	58.26673425	0.04427	0.001305652
1.0	0.999049548	3.6030E + 27	3.299E + 25	1.0921E + 02	4.693317626	58.75824672	0.04380	0.000936223
1.5	0.915914843	7.0630E + 27	6.933E + 25	1.0188E + 02	4.623747456	59.50091994	0.04400	0.000569496
2.0	0.850176452	1.1770E + 28	1.225E + 26	9.6082E + 01	4.56519817	60.07015326	0.04507	0.000392463
5.0	0.61553229	7.2540E + 28	9.974E + 26	7.2729E + 01	4.28674152	62.16719412	0.05418	0.000101858
10.0	0.437881262	3.4998E + 29	6.584E + 27	5.3156E + 01	3.973233547	64.05443997	0.07115	3.12378E - 05
50.0	0.145430396	2.2275E + 31	1.194E + 30	1.8660E + 01	2.926407246	69.25461052	0.19862	1.23958E - 06
100.0	0.079536155	1.5571E + 32	1.491E + 31	1.0441E + 01	2.345715332	71.77982634	0.35589	2.54148E - 07

TABLE III: Q-ball properties for $m_s = 100$ GeV, $K = -0.1$ and $M_m = 10^{14}$ GeV.

If E/Q is less than the MSSM-LSP mass, Q-balls can decay only via a squark annihilation process to SM quarks. Such Q-balls can in general evade BBN constraints on MSSM-LSP decay even for large gravitino mass. Therefore $d = 6$ AD baryogenesis can generally be compatible with BBN. In this case there is no source of gravitino dark matter, therefore an alternative candidate is necessary. This should be produced at a low enough temperature to evade dilution due to the low reheating temperature. An axion is a good candidate in this case.

In general, gravitino dark matter in the MSSM requires either gravitinos from thermal scattering or from non-thermal MSSM-LSPs which are produced at low temperature, below the freeze-out temperature of the MSSM-LSPs. Unstable Q-balls provide a natural source of non-thermal MSSM-LSPs in the gauge-mediated MSSM with AD baryogenesis, since in the case of $d = 6$ AD baryogenesis the Q-balls typically decay at $T_d \lesssim 10$ GeV.

If E/Q is larger than the MSSM-LSP mass, the Q-balls will decay to MSSM-LSPs. To have decaying non-thermal MSSM-LSPs which are compatible with nucleosynthesis, the gravitino must be sufficiently light, $m_{3/2} \lesssim 1$ GeV for the case of stau or sneutrino MSSM-LSPs. This requires that the AD condensate fragments to both positive and negative charged Q-balls, since in the case where only positive charged Q-balls are produced the gravitino mass is fixed by B-conservation and R-parity conservation to be $m_{3/2} \approx 2$ GeV. In GMSB the AD condensate is elliptical, which results in positive and negative charged Q-balls in numerical simulations. Therefore the factor of two enhancement of the MSSM-LSP density necessary to have $m_{3/2} \lesssim 1$ GeV is likely to be naturally achieved.

In this case decaying Q-balls in $d = 6$ AD baryogenesis provides a remarkably minimal model for baryogenesis and gravitino dark matter in the gauge-mediated MSSM, requiring now new fields or interactions beyond the MSSM.

We note that in the case where gravitino dark matter and the baryon asymmetry both originate from Q-ball decay, there could be some unique signatures which could distinguish the model from a generic non-thermal source of gravitino dark matter. In particular, correlated baryon and dark matter isocurvature perturbations can be naturally generated in this class of model, due to phase fluctuations of the AD field [25–27].

An alternative model for gravitino dark matter from Q-ball decay has been proposed in [23]. This is based on Q-balls with E/Q less than the MSSM-LSP mass, so that the main decay mode is to nucleons, with a very small branching ratio to gravitinos⁶. By having a sufficiently elliptical AD condensate, it is argued that a large number of \pm Q-balls can be produced, enhancing the gravitino density relative to nucleons enough to account for gravitino dark matter. The Q-ball solutions we have discussed here should also be relevant to the study of that model also. It will be interesting to compare the model of [23] with ours in a complete study of the evolution of the AD condensate and its fragmentation.

In order to fully understand how the new Q-ball solutions and their decay modes affect $d = 6$ AD baryogenesis in GMSB and the possibility of gravitino dark matter, a global analysis which follows the evolution of the AD condensate from the beginning of its oscillations through fragmentation, Q-ball formation and decay is necessary. We will present such an analysis in a future work.

⁶ A model for gravitino dark matter from Q-ball decay has been proposed in [28]. However, this model assumes a large branching ratio to gravitinos when E/Q is less than the MSSM-LSP mass, rather than the realistic small branching ratio discussed in [23].

Acknowledgements

The work of JM is supported by the Lancaster-Manchester-Sheffield Consortium for Fundamental Physics under STFC grant ST/J000418/1.

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